ADAPTIVE IDENTIFICATION OF NONLINEAR MODELS FOR DATA STORAGE AND COMPRESSION

T. Schimming[†], H. Dedieu[†], M. Ogorzalek[‡]

[†] *Circuits and Systems Group, EPFL, Lausanne, Switzerland*
Thomas.Schimming@epfl.ch
[‡] *University of Mining and Metallurgy, Krakov, Poland*

Abstract— We propose an adaptive identification scheme for a low-dimensional nonlinear model of the human heart's ECG dynamics. We show that this scheme is suitable for data compression and possibly the detection of diagnostically significant features. Tests on real clinically measured ECG signals confirm a very good performance of the model in terms of modelling error and compression ratio.

I. INTRODUCTION

The identification of nonlinear models is of great interest in various fields including biology, medicine and economics. Here we consider a model for the Electrocardiogram (ECG), a recording (measurement) of the electrical activity generated by the heart carried out using sensors positioned on the body surface, which will be identified. The analysis of the ECG signal provides the most common non-invasive method to diagnose cardiac disfunctions.

As in most data storage and transmission applications, a well performing compression scheme is essential for achieving a good storage and transmission efficiency.

The compression aspect can be connected with the analysis aspect by using a well adapted signal model, that ideally describes the signal by a small number of meaningful parameters, thus achieving both compression and feature analysis. In the past, a number of parametric and non-parametric methods to analyze and compress ECG signals have been proposed [2-7].

Starting from a timeseries analysis of the ECG signal, we propose in this paper a low-dimensional nonlinear signal model, and we show that

- the simple time domain model can accurately describe the dynamics of the ECG
- from a modeling perspective, the ECG cycle is composed of a P, QRS and a T-wave
- a relatively small number of parameters is sufficient to encode one ECG cycle

In the following we present the results of the time series analysis, the proposed ECG time domain model, discuss the identification of it's parameters and assess the compression ratio performance of the model.

II. ECG TIME SERIES ANALYSIS

An important aspect for the modeling is always the dimension of the system that generated the time series to be modeled. A common way to estimate the dimension is to perform an embedding of sufficiently high order and to estimate topological invariants on this reconstructed state space, such as one of various dimensions, usually the correlation dimension.

The time series analysis was performed on ECG data sampled at 500 Hz from patients in resting condition. This ensures a certain stationarity of the data, which is necessary since a large number of samples are needed for reliable estimates of the correlation dimension.

We investigated different embedding techniques (derivative embedding, time delay embedding), of which we had best results with a 3-dimensional time delay embedding with a lag $\Delta\tau$ around 6 ms (Fig. 1). Note that while for larger lags the inner part of the state space unfolds better, the QRS complex will fold back for $\Delta\tau > 6ms$.

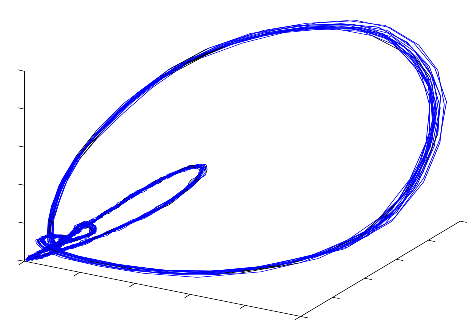

Figure 1: 3-dimensional time delay embedding of ECG data using $\Delta\tau = 6ms$

The state space reconstruction shows three loops corresponding to the three waves of the ECG, the P, QRS and T. We will see later in the proposed model a one to one correspondence of the parts of the model with each of these waves of the ECG.

A method similar to the time delay embedding shown here has been known for quite some time by the name of Vectrocardiogram [5], where the time delay is in fact achieved by measuring the ECG at different locations, thus introducing delay caused by the propagation of the waves. Of course, the results are slightly different, since the additional delay (depending on the properties of the tissue) is not necessarily constant.

III. ECG TIME DOMAIN MODEL

Inspired by the three distinguishable components of the ECG state space reconstruction we propose to describe the ECG as a superposition of three submodels, whose form is related to the underlying solution of a wave propagation problem. We chose Gaussian masking functions modulated by cosinusoides, each masking function being centered approximately around each of its corresponding wave (P, QRS, T).

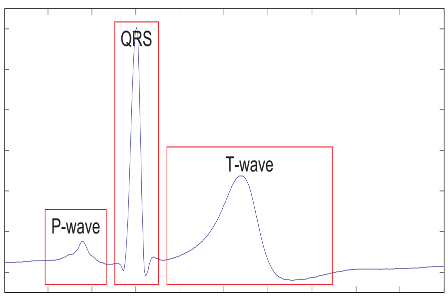

Figure 2: P, QRS and T wave of an ECG cycle

We found that this form of the submodel is applicable for each of the three waves, but the parameters differ significantly, especially for the Gaussian masking function.

The proposed model is of the form

$$x(t) = \sum_{n=1}^{3} e^{\left(-\frac{t-t_{1n}}{T_n}\right)^2} \sum_{k=0}^{K} a_{nk} \cos(k\omega_n(t-t_{2n})) \; + \; O(t) \tag{1}$$

In this model, $n = 1$ is related to the P wave, $n = 2$ to the QRS wave while $n = 3$ is related to the T wave. K is the number of harmonics needed. From experiments, $K = 2 \ldots 3$ was found to be sufficient.

As stressed before, based on this model it makes no sense to distinguish a Q, R and S wave, as is done frequently in ECG interpretation. Instead, all three belong to the same term of the model ($n = 2$).

Instead of the cosinusoides used here, other orthogonal series expansions can be considered [8, 9] for the modulation term, however, we expect that the frequency ω_n has an interpretable meaning in the sense of a fundamental frequency that other approximations can not provide.

The model was tested on a proprietary database. First, a beat detection was performed in order to identify the R-R intervals. Second, an optimization in terms of the least square error was used to fit the model parameters.

IV. MODEL IDENTIFICATION

Several approaches for identifying the parameters of (1) where evaluated, including identifying each wave separately (exploiting the strong masking of the Gaussian) and processing an entire ECG cycle at once.

It showed that performance of the identification, specifically the convergence not just towards a local optimum strongly depends on a good initial guess of the model parameters. For that, the parameters of the Gaussians are approximated by the use of the peak and corresponding 60% values of the ECG cycle (Fig. 3).

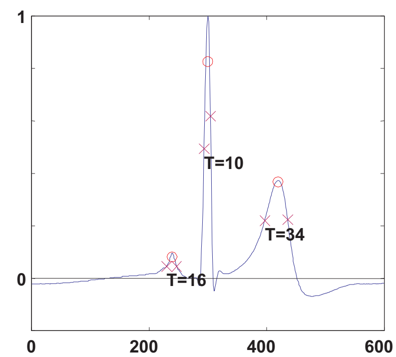

Figure 3: Peaks and 60%-points (for initial guess)

While for the identification of each wave separately the problem is easier (env. 10 parameters), the fact that the three waves overlap creates considerable problems. That's why we chose to identify all parameters simultaneously, using a conventional Levenberg-Marquand procedure.

The Levenberg-Marquard procedure is essentially a 2nd order optimization method. Let $\underline{\theta}$ be the vector of the model's parameters

$$\underline{\theta} = [t_1n, t_2n, \omega_n, T_n, a_{nk}, k = 0 \ldots K, n = 1 \ldots 3]. \tag{2}$$

and let $F(\underline{\theta})$ be defined as the approximation error vector. Let us define $C(\underline{\theta})$ as the associated mean quadratic error such that

$$F(\underline{\theta}) = [x_{\underline{\theta}}(n\Delta T) - d(n\Delta T), n = 1 \ldots L]^T \tag{3}$$

$$C(\underline{\theta}) = \|F(\underline{\theta})\|^2 \tag{4}$$

where $\Delta T = 1/f_s$ is the sampling period of the ECG signal $d(t)$ we want to approximate. For finding a vector $\hat{\underline{\theta}}$ that minimizes $C(\underline{\theta})$ (4), a modified quasi-Newton method (Levenberg-Marquardt) is used, providing a recursive estimate

$$\hat{\underline{\theta}}(k) = \hat{\underline{\theta}}(k-1) - [\underline{J}^T(k)\underline{J}(k) + \lambda_k I]^{-1}\underline{J}(k) \qquad (5)$$

which converges towards a local minimum of $C(\underline{\theta})$, $\underline{J}(k)$ is the Jacobian matrix of $F(\underline{\theta})$ and λ_k biases the direction of the descent towards the gradient, making the procedure more robust than a Gauss-Newton method. For more details of the implementation, see [10].

For the next ECG cycle we do not need to use an initial guess as crude as described before, since we can exploit the fact that the waveform does not change significantly from one ECG cycle to the next. Thus the previously estimated set of parameters can be used as a starting point of the optimization.

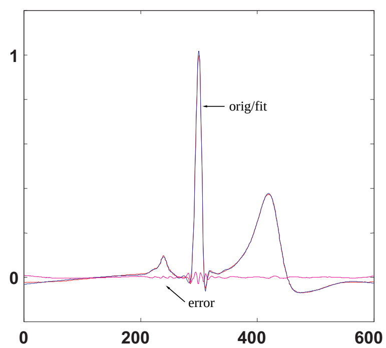

Figure 4: Original, fit and error of an entire cycle

Figure 4 shows one typical result of curve fitting using the proposed model.

Note that the quality of the fit is very high, the original waveform and the one obtained using the proposed model are hardly distinguishable. Due to the chosen harmonic series model (Eq. 1) for the modulation term, the error waveform has a cosinusoidal shape. A better insight on the model's approximation quality can be gained from the magnified view of the separate waves in Fig. 5,6,7.

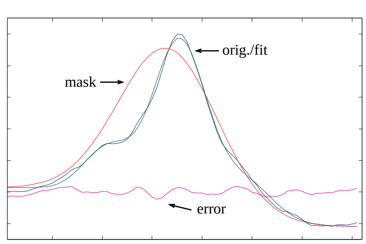

Figure 5: Orig., fit, error and mask (P wave)

Note that the relative error is larger for the P wave since it's amplitude is small compared to the QRS's. The baseline shift in Fig. 5 is due to the overlap of the models for the three waves.

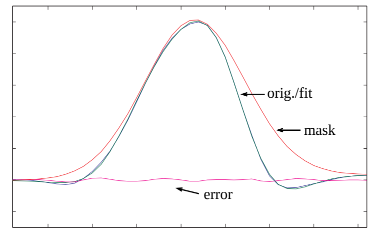

Figure 6: Orig., fit, error and mask (QRS wave)

The QRS wave (Fig. 6) presents the main motivation for the proposed model. Especially here it was found that only a Gaussian masking function is strong enough to follow exactly the well-defined onset and end of the QRS. Similar results have been found in [9].

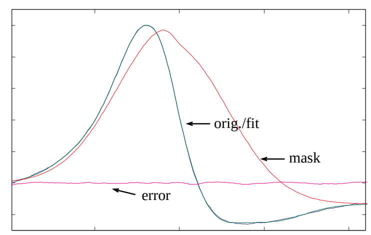

Figure 7: Orig., fit, error and mask (T wave)

Here the performance of the model is shown on noise-reduced ECG data (Fig. 2,3,4), but tests on more noisy (untreated) ECG show similar results, in fact the coding of the ECG using the model also introduces a noise reduction. Of course, the resulting residual must analyzed carefully to assure that no important information is lost due to the fact that it is not contained in the model. To accomplish this, the analysis of the residual with respect to its distribution and dimensionality seems of particular interest.

At this point, the optimization technique used is far from optimal. The *Matlab* optimization toolbox we used so far is sufficient for the model verification, but for a (desirably real time) implementation, the optimization algorithm must be improved.

This algorithm should certainly take into account the orthogonality of the modulation term's coefficients and should possibly employ a step-by-step technique by finding a raw model first and refining the model by adding higher harmonics later.

V. MODEL PERFORMANCE

Traditionally, ECG compression schemes performance is measured in a rate-distortion context and PRD (percent root mean square distortion) is usually considered.

However, similar to perceptual coding of audio and images, this error measure does not necessarily provide a good insight to the feature conservation properties of the compression scheme [1]. In the literature [1], additionally, visual inspection by experienced persons is suggested, as done in audio and image coding as well.

The data compression capabilities are essentially determined by the number of parameters that are necessary per ECG cycle. For the case of $K = 2$, which allows a high fidelity of the ECG reconstruction, 36 parameters are used. At this point no performance optimized coding (entropy code, differential encoding of the parameters to exploit small beat to beat variations) has been implemented. However, the most substantial performance gain can be expected from the differential encoding, which is possible due to the fact that the model provides meaningful parameters that are reproducible on the ECG cycle.

VI. CONCLUSION

The paper we present here shows that a very low-dimensional dynamical system provides a good model of the ECG signal. Starting from the time series analysis, a new time-domain model accounting for the ECG dynamics was constructed. In our experiments the identification of the model's parameters was feasible yielding a good approximation of the ECG cycle. In the future, additional simulations will need to be carried out using the MIT-BIH database. It must be emphasized that focusing solely on mean square error criteria does not guarantee good feature conservation. The approach using a Gaussian masking function is similar to the work in [8, 9], even though there the notion of a masking function is not used. Here we use the same type of model for all three waves of the ECG, whereas [8] applies it to the QRS only. The main difference to the approach presented there is the use of a different orthonormal basis (Hermitian polynomes) whereas here a harmonic series is used.

However, the proposed approach using a harmonic series inspires a dynamical (oscillator) model with a nonlinearity creating the higher order harmonics. This aspect makes the presented model interesting for further research in the direction of an oscillator model for the ECG.

Currently an implementation on a TMS320C3x series DSP is in development, which will allow us to further investigate the real time performance of the parameter approximation.

Concluding we would like to stress that a low-dimensional nonlinear model explaining the ECG dynamics, suitable for data compression and possibly feature detection, has been developed. Using several tests on real clinically measured ECG signals we confirmed very good performance of the model in terms of small modeling errors.

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